Topic 9: Wave phenomena - AHL
9.3 – Interference

**Topic 9.3** is an extension of Topic 4.4.

**Essential idea:** Interference patterns from multiple slits and thin films produce accurately repeatable patterns.

**Nature of science:** (1) Curiosity: Observed patterns of iridescence in animals, such as the shimmer of peacock feathers, led scientists to develop the theory of thin film interference. (2) Serendipity: The first laboratory production of thin films was accidental.
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**Understandings:**
- Young’s double-slit experiment
- Modulation of two-slit interference pattern by one-slit diffraction effect
- Multiple slit and diffraction grating interference patterns
- Thin film interference
Applications and skills:
• Qualitatively describing two-slit interference patterns, including modulation by one-slit diffraction effect
• Investigating Young’s double-slit experimentally
• Sketching and interpreting intensity graphs of double-slit interference patterns
• Solving problems involving the diffraction grating equation
• Describing conditions necessary for constructive and destructive interference from thin films, including phase change at interface and effect of refractive index
• Solving problems involving interference from thin films
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Guidance:
• Students should be introduced to interference patterns from a variety of coherent sources such as (but not limited to) electromagnetic waves, sound and simulated demonstrations
• Diffraction grating patterns are restricted to those formed at normal incidence
• The treatment of thin film interference is confined to parallel-sided films at normal incidence
• The constructive interference and destructive interference formulae listed below and in the data booklet apply to specific cases of phase changes at interfaces and are not generally true
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Data booklet reference:
• \( n\lambda = d \sin \theta \)
• Constructive interference: \( 2dn = (m + \frac{1}{2})\lambda \)
• Destructive interference: \( 2dn = m\lambda \)

Theory of knowledge:
• Most two-slit interference descriptions can be made without reference to the one-slit modulation effect. To what level can scientists ignore parts of a model for simplicity and clarity?

Utilization:
• Compact discs are a commercial example of the use of diffraction gratings
• Thin films are used to produce anti-reflection coatings
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Aims:

• **Aim 4:** two scientific concepts (diffraction and interference) come together in this sub-topic, allowing students to analyze and synthesize a wider range of scientific information

• **Aim 6:** experiments could include (but are not limited to): observing the use of diffraction gratings in spectroscopes; analysis of thin soap films; sound wave and microwave interference pattern analysis

• **Aim 9:** the ray approach to the description of thin film interference is only an approximation. Students should recognize the limitations of such a visualization
PRACTICE: Two apertures in a sea wall produce two diffraction patterns as shown in the animation.

(a) Which letter represents a maximum displacement above equilibrium? __C__ (crest-crest)
(b) Which letter represents a maximum displacement below equilibrium? __B__ (trough-trough)
(c) Which letter represents a minimum displacement from equilibrium? __A__ (crest-trough)

- Crest-crest = max high. Trough-trough = max low.
- Crest-trough = minimum displacement.
Double-slit interference

• Double-slit interference with light waves was explored by Thomas Young in 1801.

• The following formula, as you may recall from Topic 4.4, relates wavelength $\lambda$, slit separation $d$, and distance $D$ to screen, to the separation $s$ of the maxima:

$$s = \frac{\lambda D}{d}$$

Young’s double-slit experiment
**Double-slit interference**

- This animation of water waves shows single-slit diffraction and two-slit interference over a large field.
- Where two crests or two troughs meet there is constructive interference.
- Where a crest meets a trough there is destructive interference.
Double-slit interference

EXAMPLE: Coherent light having a wavelength of 675 nm is incident on an opaque card having two vertical slits separated by 1.25 mm. A screen is located 4.50 m away from the card. What is the distance between the central maximum and the first maximum?

SOLUTION: Use \( s = \frac{\lambda D}{d} \).

- \( \lambda = 675 \times 10^{-9} \) m, \( D = 4.50 \) m, and \( d = 1.25 \times 10^{-3} \) m.
- Thus

\[
s = \frac{\lambda D}{d} = \frac{675 \times 10^{-9} \times 4.50}{1.25 \times 10^{-3}} = 0.00243 \text{ m.}
\]
Double-slit interference – intensity

• Because each slit in two-slit interference also acts like a single slit, likewise introduced in Topic 4, the amplitude will be modulated.

• The bottom image shows the double-slit pattern. Note the regular separation $s$ but the varying intensity.

• The top image shows the interference pattern if one of the two slits is blocked. In fact, the “single-slit intensity envelope” determines the double-slit intensity pattern.
Double-slit interference – intensity

- If the double-slit intensity were not modulated by the single-slit effect, it would look like this:
- Because of single-slit constructive and destructive interference, the actual double-slit intensity pattern looks like this:
Incidentally, the overall double-slit intensity of the brightest portions of the interference pattern are brighter than those of the single-slit because there are two sources, rather than one.

It turns out that if \( N \) is the number of slits, and \( I_1 \) is the intensity of the single-slit central maximum, then

\[
I_N = N^2 I_1
\]

intensity of central maximum for \( N \) slits
Multiple-slit interference

- The separation between maxima $s = \frac{\lambda D}{d}$ that we calculated for two-slit interference shows the location of the single-slit envelope peaks for multiple slits separated by $d$, as illustrated:

The single-slit envelope still applies.
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*Multiple-slit interference*

- The intensities for 1, 2, 3, 4 and 5 equally-spaced slits are shown here, for part of the central maximum envelope.
- Note the double-slit separation $s = \frac{\lambda D}{d}$ remains the same for multiple slits.
- Also observe that the intensity increases with the increase in slits. More light gets through.

<table>
<thead>
<tr>
<th>Slits</th>
<th>Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 slit</td>
<td>$I_2$</td>
</tr>
<tr>
<td>2 slits</td>
<td>$(9/4)I_2$</td>
</tr>
<tr>
<td>3 slits</td>
<td>$(16/4)I_2$</td>
</tr>
<tr>
<td>4 slits</td>
<td>$(25/4)I_2$</td>
</tr>
<tr>
<td>5 slits</td>
<td>primary maxima</td>
</tr>
</tbody>
</table>

secondary maxima

$\lambda$: wavelength, $D$: distance between the slits, $d$: slit width.
EXAMPLE: A light having a wavelength of $\lambda = 675 \times 10^{-9}$ m is projected onto four vertical slits equally spaced at 16.875 $\mu$m between slits. The slit widths are all 6.75 $\mu$m. The resulting diffraction pattern is projected on a wall that is 5.00 m away from the slits.

(a) Find the separation between the bright points in the pattern.

SOLUTION: Use $s = \lambda D / d$.

- $\lambda = 675 \times 10^{-9}$ m, $D = 5.00$ m, and $d = 1.45 \times 10^{-6}$ m.
- Thus

$$s = \frac{\lambda D}{d}$$

$$= 675 \times 10^{-9} \times 5.00 / 16.875 \times 10^{-6} = 0.200 \text{ m.}$$
EXAMPLE: A light having a wavelength of $\lambda = 675 \times 10^{-9}$ m is projected onto four vertical slits equally spaced at 16.875 $\mu$m between slits. The slit widths are all 6.75 $\mu$m. The resulting diffraction pattern is projected on a wall that is 5.00 m away from the slits.

(b) Determine the width of the brightest central region of the overall pattern.

SOLUTION: Use $\theta = \lambda / b$.

- $\lambda = 675 \times 10^{-9}$ m, and $b = 6.75 \times 10^{-6}$ m.
- Thus $\theta = \lambda / b = 675 \times 10^{-9} / 6.75 \times 10^{-6} = 0.100$ rad.
- The central region is twice this, or $\theta = 0.200$ rad.
- Finally, $width = D\theta = 5.00(0.200) = 1.00$ m.
EXAMPLE: A light having a wavelength of $\lambda = 675 \times 10^{-9}$ m is projected onto four vertical slits equally spaced at 16.875 $\mu$m between slits. The slit widths are all 6.75 $\mu$m. The resulting diffraction pattern is projected on a wall that is 5.00 m away from the slits.

(c) Approximately how many bright points will fit in this central region?

SOLUTION:

- We have a total width of 1.00 m over which to distribute bright points having a separation of 0.200 m.
- Thus $N = 1.00 / 0.200 = 5.00$ points.
EXAMPLE: A light having a wavelength of $\lambda = 675 \times 10^{-9}$ m is projected onto four vertical slits equally spaced at 16.875 $\mu$m between slits. The slit widths are all 6.75 $\mu$m. The resulting diffraction pattern is projected on a wall that is 5.00 m away from the slits.

(d) Sketch the pattern’s intensity in the region of the central maximum.

SOLUTION:

- Because we have four slits, there will be two small maxima between each large one. All will be constrained to fit within the single-slit envelope.
The diffraction grating

- Diffraction gratings are used to make optical spectra.

Light source

Continuous spectrum

Cool gas X

Absorption spectrum

Compare...

Hot gas X

Emission spectrum

Same fingerprint!
The diffraction grating

- A typical diffraction grating consists of a large number of parallel, equally-spaced lines or grooves, etched in a glass or plastic substrate through which light passes, or is reflected from.

- Different wavelengths are diffracted at different angles producing interference maxima at angles $\theta$ given by

$$n\lambda = d \sin \theta$$

where $n$ is the order of the maxima. $n = 0$ is the central maximum, $n = 1$ is on either side of the central maximum, etc.
EXAMPLE: A diffraction grating that has 750 lines per millimeter is illuminated by a monochromatic light which is normal to the grating. A third-order maximum is observed at an angle of 56° to the straight-through direction. Determine the wavelength of the light.

SOLUTION: Use \( n\lambda = d\sin\theta \).

- \( n = 3 \), and \( \theta = 56° \).
- \( d \) must be calculated:
  \[
  N = 750 \text{ lines} / 1.00 \times 10^{-3} \text{ m} = 750000 \text{ lines m}^{-1}.
  \]
  \[
  d = 1 / N = 1 / 750000 = 1.33 \times 10^{-6} \text{ m}.
  \]
- \( \lambda = d\sin\theta / n = (1.33 \times 10^{-6}) \sin 56° / 3 
  = 3.68 \times 10^{-7} \text{ m} = 368 \text{ nm} \).
Thin film interference

- Soap bubbles and gasoline spills on water, and iridescence in insects and feathers, are all examples of thin film interference.
Before we can understand how this interference works we need to review how waves are reflected at the boundary of a heavy rope and a light rope. A single pulse is sent from left to right, as shown.

- Note that the reflection is IN PHASE and the transmission is also IN PHASE.

Note that $v$ is less in the heavy rope than it is in the light rope.
We can remark on the following three properties:

1. If a wave enters a boundary from a medium where the wave speed is lower, it will reflect IN PHASE.
2. If a wave enters a boundary from a medium where the wave speed is higher, it will reflect OUT OF PHASE.
3. In any case, the wave is always transmitted through the boundary IN PHASE with the original pulse.
Thin film interference

- Consider a thin film of thickness $d$ made of a transparent medium having an index of refraction $n$.
- Light having a wavelength $\lambda$ is normally incident on the film. We show it at a slight angle so that you can see reflection and refraction clearly…
- At the top surface, some light is reflected, and some is refracted…
- At each subsequent surface reflection and refraction occur…
Thin film interference

- The speed of light in air is \( c \), and the speed of light in the medium is \( v = \frac{c}{n} \) which is always less than \( c \).
- At the TOP SURFACE the reflected ray is OUT OF PHASE with the incident ray. (See the ropes…)
- Inside the film the transmitted ray is IN PHASE with the incident ray.
- At the BOTTOM SURFACE the reflected ray is IN PHASE with the incident wave.
- At the TOP SURFACE the phase of the transmitted ray is NOT CHANGED during transmission.
**Thin film interference**

- The distance the light travels INSIDE the film is \(2d\). The angle of incidence is actually 0° and has been exaggerated.
- The speed of light in the medium is \(c/n\).
- From \(\text{speed} = \text{distance} / \text{time}\) we see that inside the medium the time for a typical crest to make it through the medium is
  \[
  \text{time} = \text{distance} / \text{speed} = 2d / (c/n) = 2dn / c.
  \]
- But outside the medium a crest travels \(m\) wavelengths \(\lambda\) in
  \[
  \text{time} = \text{distance} / \text{speed} = m\lambda / c.
  \]
Thin film interference

- Because the light in Ray A and Ray B are half a wavelength out of phase, when the two rays rejoin at the top surface, they will destructively interfere if their transit times are equal:

\[ \text{time} = \frac{2dn}{c} = \frac{m}{c}. \]

\[ 2dn = m\lambda \] thin film destructive interference

- Similarly, it can be deduced that the rejoining rays will constructively interfere if the following relation holds:

\[ \text{time} = \frac{2dn}{c} = \frac{(m + \frac{1}{2})}{c}. \]

\[ 2dn = (m + \frac{1}{2})\lambda \] thin film constructive interference
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Thin film interference

- These formulas were derived assuming the observer was on the same side as the light source.
- These formulas were derived assuming that only one of the rays was reflected $\pi$ out of phase.
- If both rays are $\pi$ out of phase, reverse the formulas.

PRACTICE: Predict what you will see if you view the film from the side away from the source.

SOLUTION: Ray A will be in phase with the incident light. Because $n > 1$, both reflections within the medium will be in phase. Thus Ray B is also in phase. Just reverse the formulas.
EXAMPLE: Explain why thin-film interference in a vertical soap bubble looks like this.

SOLUTION: Because of gravity, the film is thickest at the bottom and thinnest at the top.

- Because of the varying thicknesses, different wavelengths of light are both constructively and destructively interfered with, producing the different width color bands.
- At the top the color is black because the thickness of the film is negligible to the wavelength, producing complete destructive interference.
PRACTICE: A film of oil having a refractive index of 1.40 floats on a puddle of rain water having a refractive index of 1.33. The puddle is illuminated by sunlight. When viewed at near-normal incidence a particular region of the oil film has an orange color, corresponding to a wavelength of 575 nm.

(a) Explain how the refractive indices of the air, oil, and water all play a part in producing this orange color.

SOLUTION: Since $n_{\text{air}} < n_{\text{oil}}$, at the air-oil boundary the light is reflected out of phase. Since $n_{\text{oil}} > n_{\text{water}}$, at the oil-water boundary the light is reflected in phase. Since we see orange, we know that the interference is constructive for 575 nm. Thus $2dn = (m + \frac{1}{2})\lambda$. 
PRACTICE: A film of oil having a refractive index of 1.40 floats on a puddle of rain water having a refractive index of 1.33. The puddle is illuminated by sunlight. When viewed at near-normal incidence a particular region of the oil film has an orange color, corresponding to a wavelength of 575 nm.

(b) Calculate the possible thicknesses of the film in the orange region.

SOLUTION:

As determined on the previous slide $2dn = (m + \frac{1}{2})\lambda$, where $n = 1.40$ is the refractive index of the oil.

$$d = \frac{(m + \frac{1}{2})\lambda}{2n} = \frac{(m + \frac{1}{2})\times575 \text{ nm}}{2\times1.40} = (m + \frac{1}{2})\times205 \text{ nm}, \text{ for } m = 0, 1, 2, \ldots$$
PRACTICE: A film of oil having a refractive index of 1.40 floats on a puddle of rain water having a refractive index of 1.33. The puddle is illuminated by sunlight. When viewed at near-normal incidence a particular region of the oil film has an orange color, corresponding to a wavelength of 575 nm.

(c) Calculate the minimum thickness of the film in the orange region.

SOLUTION:
• Just substitute \( m = 0 \) into your solution:
  \[ d = (0 + \frac{1}{2}) \times 205 \text{ nm} = 103 \text{ nm}. \]
EXAMPLE: Magnesium fluoride MgF$_2$ has $n = 1.37$. If applied in a thin layer over an optical lens made of glass having $n = 1.38$, what thickness should it be so that light having a wavelength of 528 nm is not reflected from the lens?

SOLUTION: $n_{\text{air}} < n_{\text{MgF$_2$}}$, so at the air-MgF$_2$ boundary the light is reflected out of phase. $n_{\text{MgF$_2$}} < n_{\text{lens}}$, so at the MgF$_2$-lens boundary the light is again reflected out of phase. Thus BOTH reflections are IN PHASE.

- Since we desire destructive interference, we need the thickness of the MgF$_2$ layer to satisfy $2dn = (m + \frac{1}{2})\lambda$.

- For $m = 0$ we have
  
  $$d = (0 + \frac{1}{2})\lambda / 2n = \lambda / 4n = 528 \text{ nm} / 2 \times 1.37 = 193 \text{ nm}.$$